

EE 330

Lecture 15

Devices in Semiconductor Processes

- Analysis of Nonlinear Circuits
- Diodes

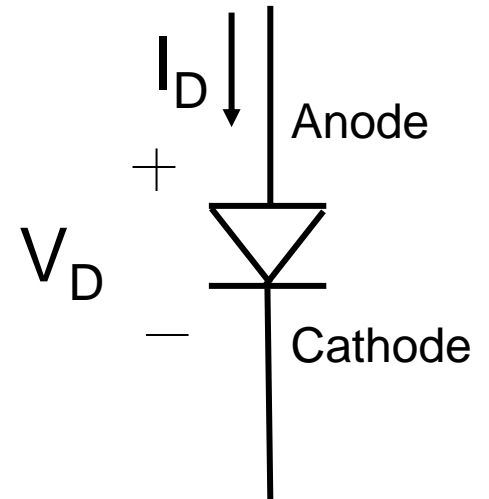
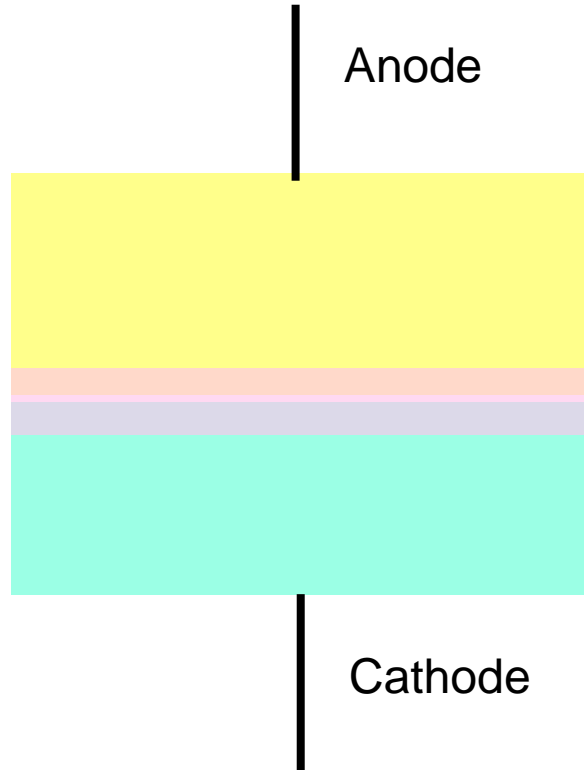
Photo courtesy of the director of the National Institute of Health (NIH)



As a courtesy to fellow classmates, TAs, and the instructor

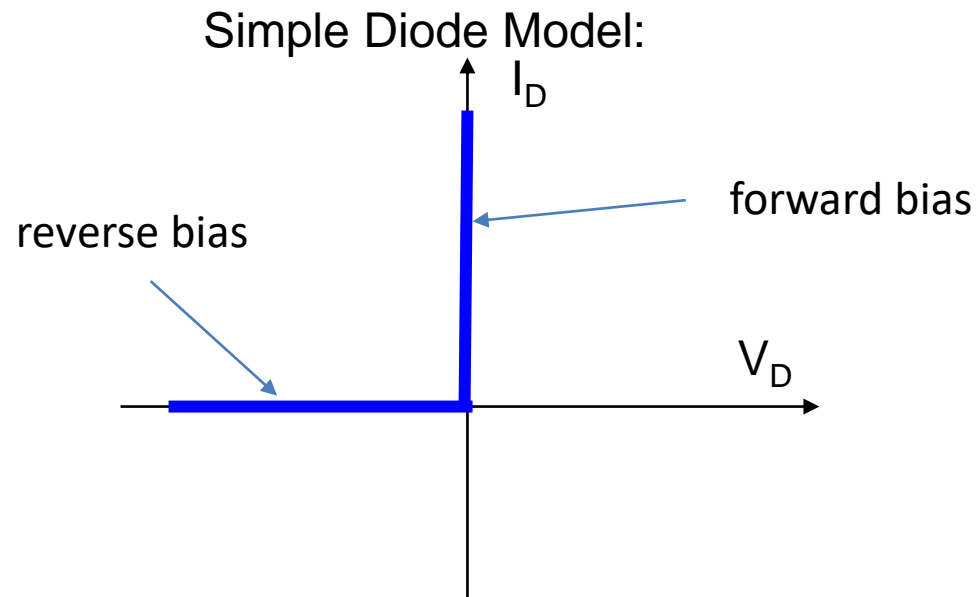
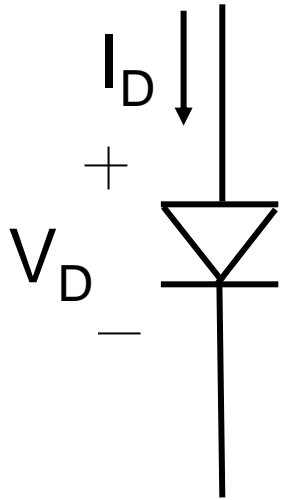
Wearing of masks during lectures and in the laboratories for this course would be appreciated irrespective of vaccination status

pn Junctions



Circuit Symbol

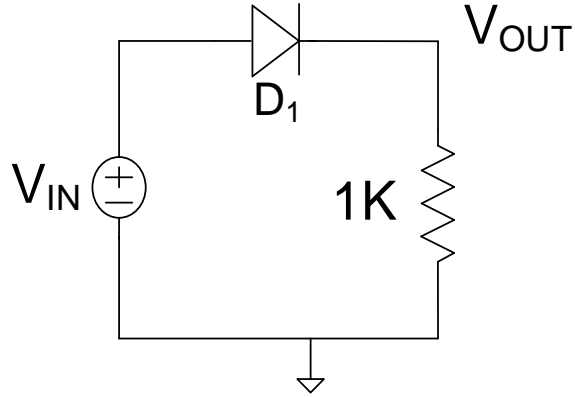
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- This is a piecewise model
- pn junction serves as a “rectifier” passing current in one direction and blocking it in the other direction

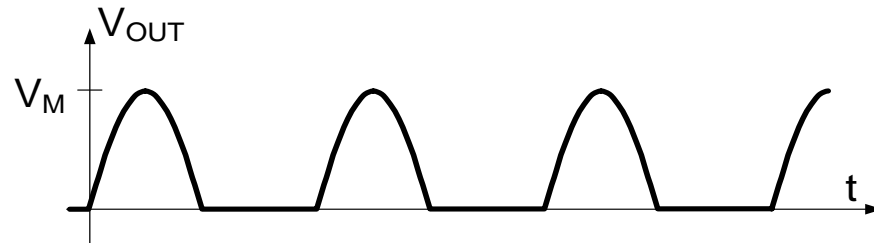
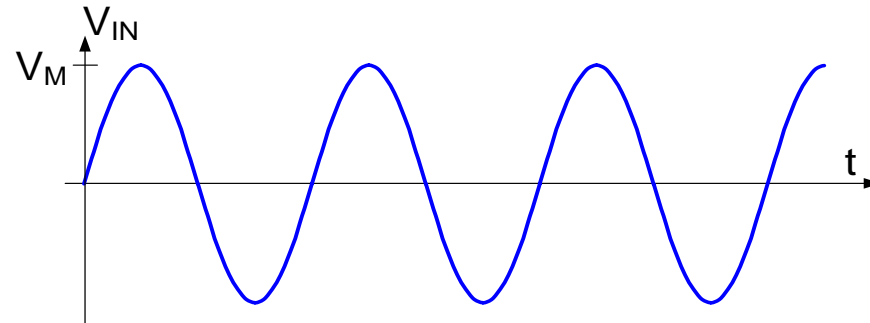
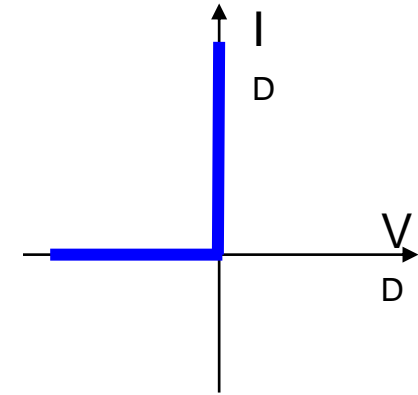
Review from last lecture

Rectifier Application:



$$V_{IN} = V_M \sin \omega t$$

Simple Diode Model:

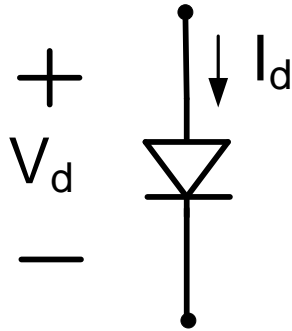


Analysis based upon “passing current” in one direction and “blocking current” in the other direction

I-V characteristics of pn junction

(signal or rectifier diode)

Improved Diode Model:

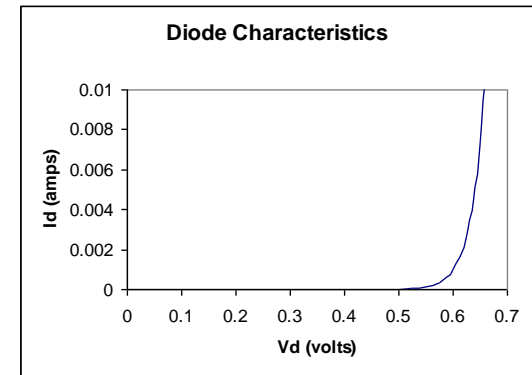


Diode Equation $I_D = I_S \left(e^{\frac{V_d}{nV_t}} - 1 \right)$
 (not a piecewise model !)

Simplification of Diode Equation:

Under reverse bias ($V_d < 0$), $I_D \cong -I_S$

Under forward bias ($V_d > 0$), $I_D = I_S e^{\frac{V_d}{nV_t}}$



I_S in 10fA - 100fA range (for signal diodes)

n typically about 1

$$V_t = \frac{kT}{q}$$

$$k/q = 8.62 \times 10^{-5} \text{ VK}^{-1}$$

V_t is about 26mV at room temp

Simplification essentially identical model except for V_d very close to 0

Diode Equation or forward bias simplification are unwieldy to work with analytically

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Diode Equation: (simplification) $I = \begin{cases} I_S A e^{\frac{V}{nV_T}} & V > 0 \\ -I_S & V < 0 \end{cases}$ forward bias reverse bias

Diode Equation: (further simplification) $I = \begin{cases} I_S e^{\frac{V}{nV_T}} & V > 0 \\ 0 & V < 0 \end{cases}$ forward bias reverse bias

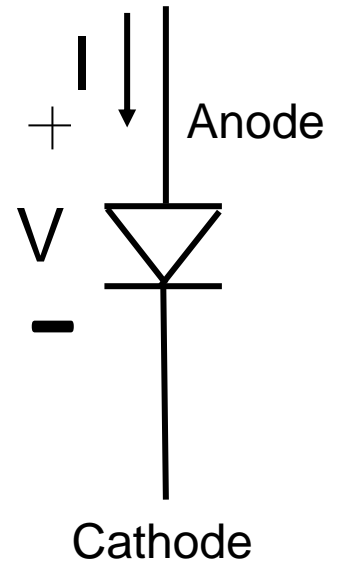
$$I_S = J_S A$$

{ J_S } is model parameter (or I_S is a model parameter if A is fixed)

{ A } is design parameter, A is the cross-sectional area of the junction

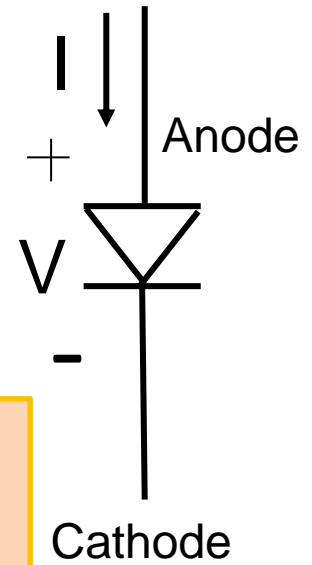
Slight discontinuity at $V=0$ in these models (which doesn't exist in real diodes) but of no consequence unless V is very close to 0

I_S is often given in data sheets and model files



Diode Model Summary

Ideal Diode Model	$V_D = 0$	$I_D > 0$	forward bias
	$I_D = 0$	$V_D < 0$	reverse bias



$$I_S = J_S A$$

Diode Equation

$$I_D = I_S \left(e^{\frac{V_d}{nV_t}} - 1 \right)$$

Diode Equation: (simplification)

$$I = \begin{cases} I_S e^{\frac{V}{nV_T}} & V > 0 \\ -I_S & V < 0 \end{cases}$$

forward bias
reverse bias

Diode Equation: (further simplification)

$$I = \begin{cases} I_S e^{\frac{V}{nV_T}} & V > 0 \\ 0 & V < 0 \end{cases}$$

forward bias
reverse bias

Little difference in these models, if any, in most applications. Typically, any referred to as the Diode Equation

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Diode Equation: (further simplification)

$$I = \begin{cases} J_S A e^{\frac{V}{nV_T}} & V > 0 \quad \text{forward bias} \\ 0 & V < 0 \quad \text{reverse bias} \end{cases}$$

$$I_S = J_S A$$

J_S (or I_S) is strongly temperature dependent

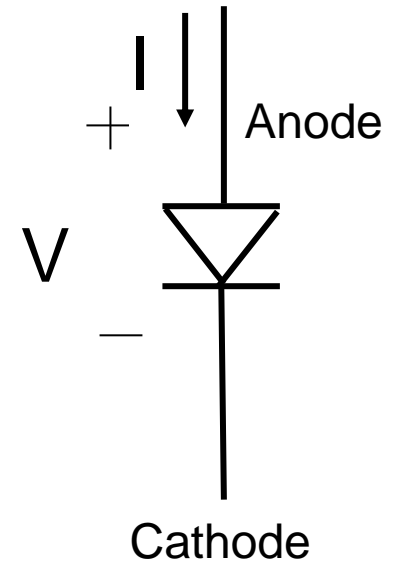
With $n=1$, for $V>0$,

$$J_S = J_{SX} T^m e^{\frac{-V_{G0}}{V_t}}$$

$\{J_{SX}, m, n\}$ are model parameters

$\{A\}$ is a design parameter

$\{T, V_{G0}, k/q\}$ are environmental parameters and physical constants



Diode Equation: (further simplification showing more detail)

$$I(T) = \begin{cases} \left(J_{SX} \left[T^m e^{\frac{-V_{G0}}{V_t}} \right] \right) A e^{\frac{V}{V_t}} & V > 0 \\ 0 & V < 0 \end{cases}$$

Typical values for key parameters: $J_{SX}=0.5A/\mu^2$, $V_{G0}=1.17V$, $m=2.3$

This simplification is a piecewise model !

Diode Equation (even simplification) unwieldy to work with analytically. **Why?**

World's simplest diode circuit

Determine V_{OUT}

Assume forward bias , simplified diode equation model

$$5 = V_D + V_{OUT}$$

$$V_{OUT} = I_D \cdot 1K$$

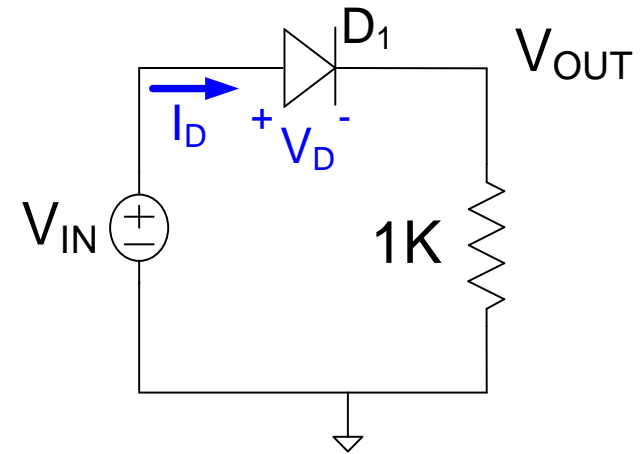
$$I_D = I_S e^{\frac{V_D}{nV_t}}$$

3 independent equations and 3 unknowns



$$V_{OUT} = I_S e^{\frac{5-V_{OUT}}{nV_t}} \cdot 1K$$

$$V_{OUT}=?$$



$$V_{IN}=5V$$

Explicit expression does not exist for V_{OUT} !

I-V characteristics of pn junction

(signal or rectifier diode)

Diode Equation

$$I_D = I_S \left(e^{\frac{V_d}{nV_t}} - 1 \right)$$

I_S often in the 10fA to 100fA range
 I_S proportional to junction area

V_t is about 26mV at room temp

Simplification of Diode Equation:

$$I_D = \begin{cases} I_S e^{\frac{V_D}{nV_T}} & V > 0 \\ -I_S & V < 0 \end{cases}$$

How much error is introduced using the simplification for $V_d > 0.5V$? (assume $n=1$)

$$\varepsilon = \frac{I_S \left(e^{\frac{V_d}{V_t}} - 1 \right) - I_S e^{\frac{V_d}{V_t}}}{I_S \left(e^{\frac{V_d}{V_t}} - 1 \right)} \quad \varepsilon < \frac{1}{\frac{0.5}{e^{.026}}} = 4.4 \bullet 10^{-9}$$

How much error is introduced using the simplification for $V_d < -0.5V$?

$$\varepsilon < e^{\frac{-0.5}{.026}} = 4.4 \bullet 10^{-9}$$

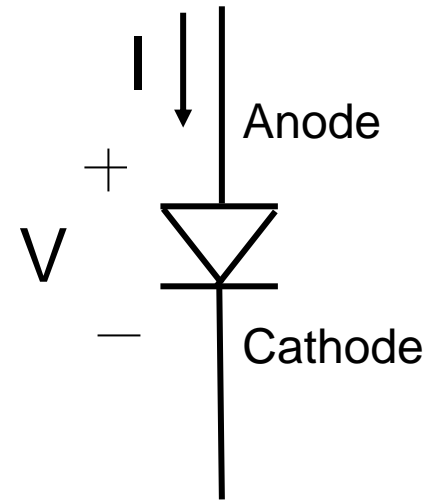
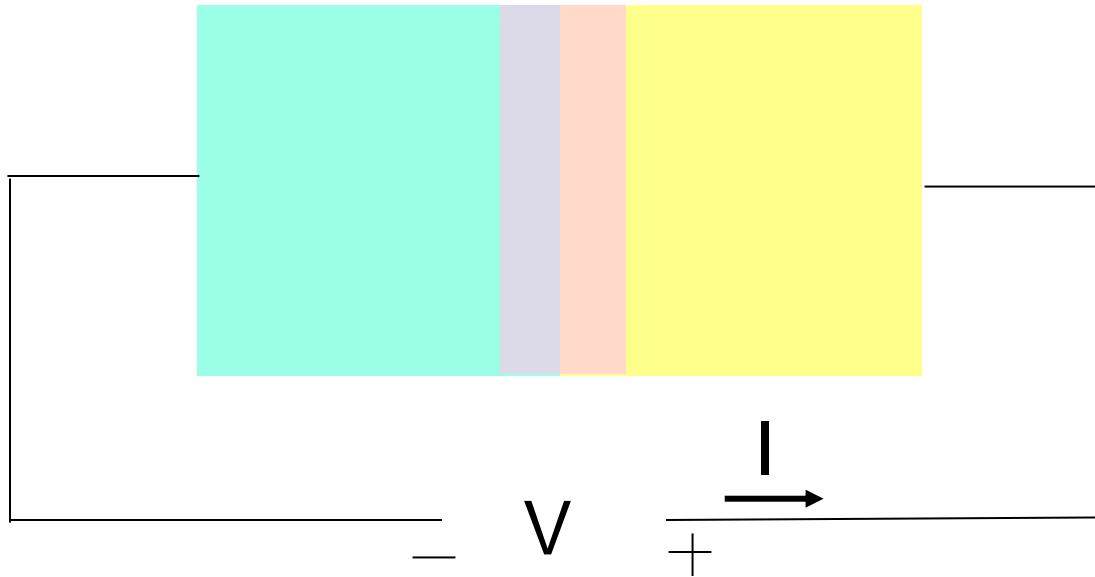
Simplification almost never introduces any significant error

Will you impress your colleagues or your boss if you use the more exact diode equation when $V_d < -0.5V$ or $V_d > +0.5V$?



Will your colleagues or your boss be unimpressed if you use the more exact diode equation when $V_d < -0.5V$ or $V_d > +0.5V$?

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“Diode Equation”:

(good enough for most applications
when ideal diode model is inadequate)

$$I = \begin{cases} J_s A e^{\frac{v}{nV_T}} & V > 0 \\ 0 & V < 0 \end{cases}$$

Note: $I_s = J_s A$

J_s = Sat Current Density (in the 1aA/u² to 1fA/u² range)

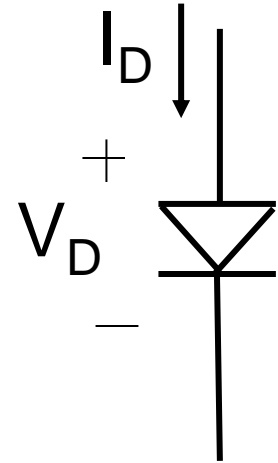
A = Junction Cross Section Area

$V_T = kT/q$ (k/q = 1.381x10⁻²³V•C/°K / 1.6x10⁻¹⁹C = 8.62x10⁻⁵V/°K)

n is approximately 1

I_S highly temperature dependent

Example: Consider diode operating under forward bias



$$I_D(T) = \left(J_{SX} \left[T^m e^{\frac{-V_{G0}}{V_t}} \right] \right) A e^{\frac{V_D}{V_t}}$$

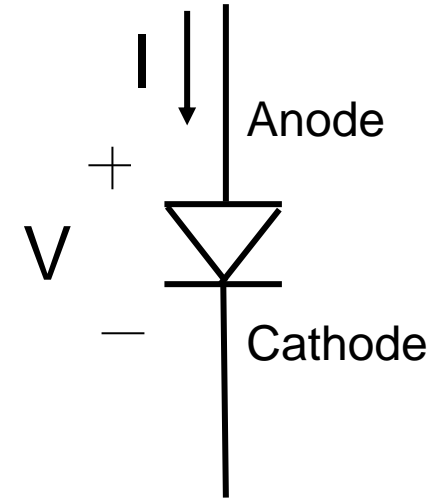
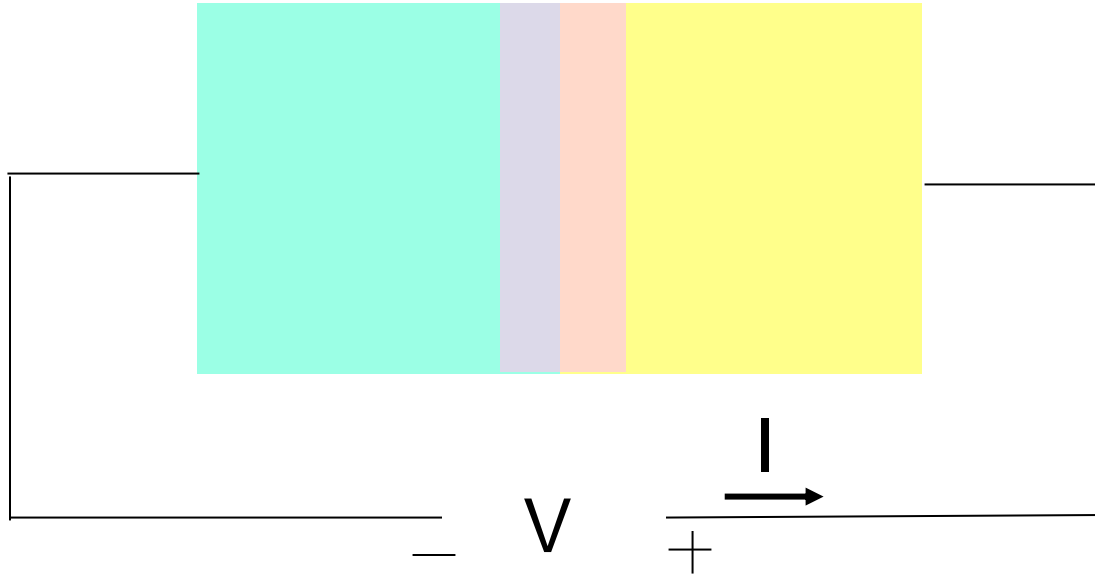
What percent change in I_S will occur for a 1°C change in temperature at room temperature?

$$\frac{\Delta I_S}{I_S} = \frac{\left(J_{SX} \left[T_{T_2}^m e^{\frac{-V_{G0}}{V_t(T_2)}} \right] \right) A e^{\frac{V_D}{V_t}} - \left(J_{SX} \left[T_{T_1}^m e^{\frac{-V_{G0}}{V_t(T_1)}} \right] \right) A e^{\frac{-V_{G0}}{V_t(T_2)}}}{\left(J_{SX} \left[T_{T_1}^m e^{\frac{-V_{G0}}{V_t(T_1)}} \right] \right) A e^{\frac{-V_{G0}}{V_t(T_2)}}} = \frac{\left(\left[T_{T_2}^m e^{\frac{-V_{G0}}{V_t(T_2)}} \right] \right) - \left(\left[T_{T_1}^m e^{\frac{-V_{G0}}{V_t(T_1)}} \right] \right)}{\left(\left[T_{T_1}^m e^{\frac{-V_{G0}}{V_t(T_1)}} \right] \right)}$$

$$\frac{\Delta I_S}{I_S} = \frac{(1.240 \times 10^{-15}) - (1.025 \times 10^{-15})}{(1.025 \times 10^{-15})} 100\% = 21\%$$

- Attempts to measure I_S in our laboratories can result in large errors !
- Most circuits whose performance depends upon precise value for I_S are not practical

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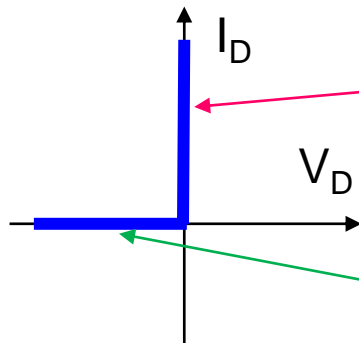


Diode Equation:
(good enough for most applications)

$$I = \begin{cases} J_s A e^{\frac{v}{nV_T}} & V > 0 \\ 0 & V < 0 \end{cases}$$

$$I_s = J_s A$$

Simple Diode Model:

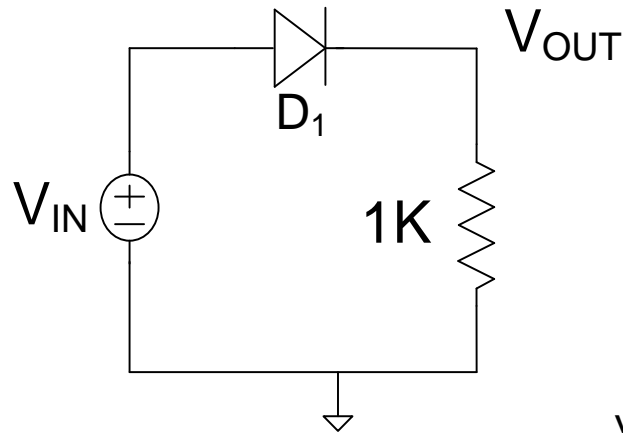


Often termed the “conducting” or “ON” state

Often termed the “nonconducting” or “OFF” state

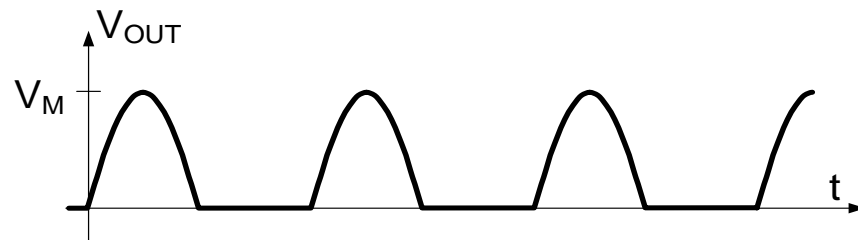
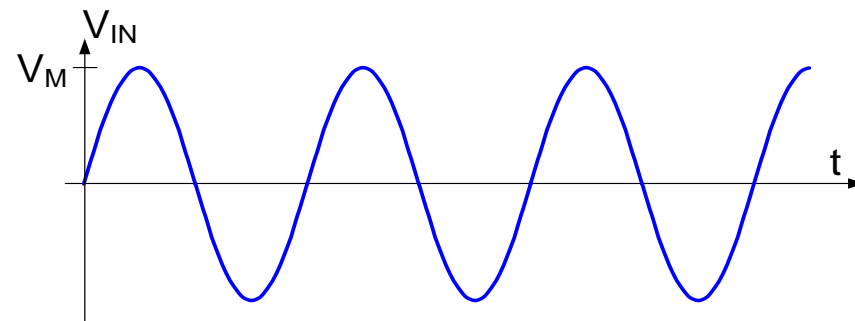
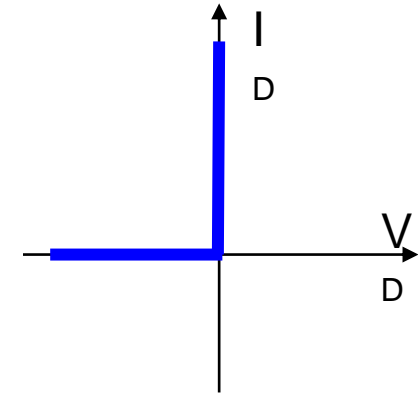
What basic circuit analysis principles were used to analyze this circuit?

Rectifier Application:



$$V_{IN} = V_M \sin \omega t$$

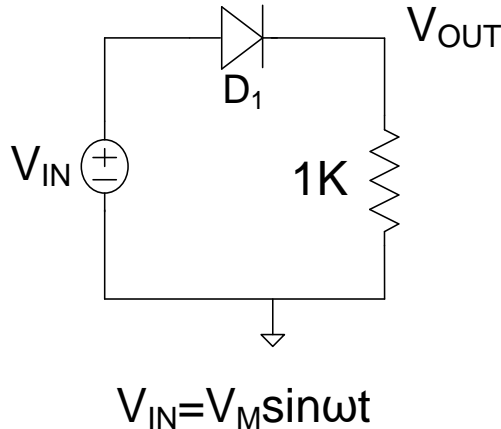
Simple Diode Model:



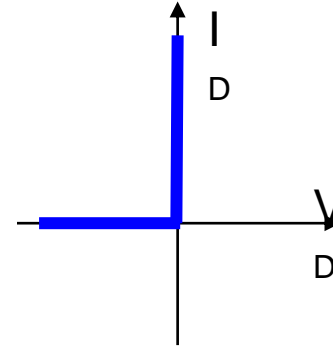
Analysis based upon “passing current” in one direction and “blocking current” in the other direction

Was the previous analysis rigorous?

Rectifier Application:



Simple Diode Model:



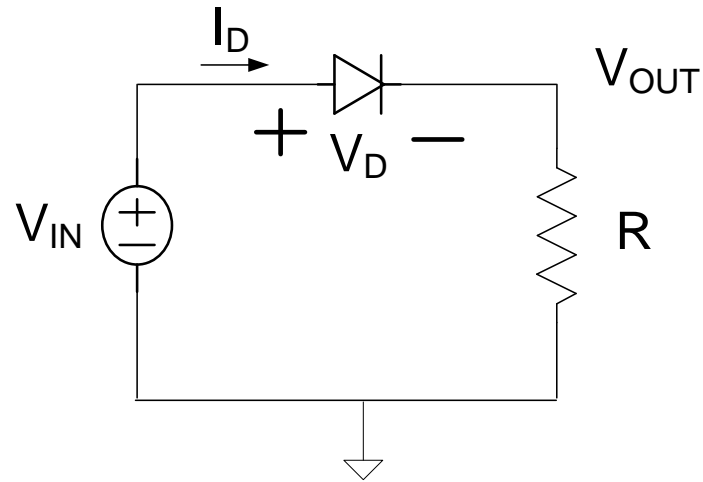
Analysis based upon “passing current” in one direction and “blocking current” in the other direction

What tools do we have for analyzing circuits?

KCL, KVL, current divider, voltage divider, superposition, Thevenin equivalent, Norton equivalent, nodal analysis, mesh analysis, passing current, blocking current

Can all of these tools be used to analyze nonlinear circuits?

Consider again the basic rectifier circuit



- Previously considered sinusoidal excitation
- Previously gave “qualitative” analysis
- **Rigorous analysis method is essential**

$$V_{OUT} = ?$$

Analysis of Nonlinear Circuits

(Circuits with one or more nonlinear devices)

What analysis tools or methods can be used?

KCL ?

Nodal Analysis ?

KVL?

Mesh Analysis ?

~~Superposition?~~

Two-Port Subcircuits ?

~~Voltage Divider ?~~

Passing Current ?

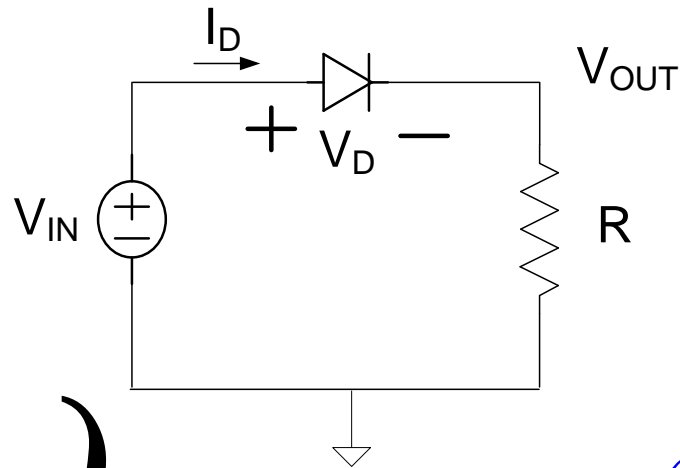
~~Current Divider?~~

Blocking Current ?

~~Thevenin and Norton Equivalent Circuits?~~

- How are piecewise models accommodated?
- Will address the issue of how to rigorously analyze nonlinear circuits with piecewise models later

Consider again the basic rectifier circuit



$$V_{IN} = V_D + I_D R$$

$$V_{OUT} = I_D R$$

$$I_D = I_S \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

$$V_{OUT} = I_S R \left(e^{\frac{V_{IN} - V_{OUT}}{V_t}} - 1 \right)$$

This analysis is rigorous (using only KVL and device models)

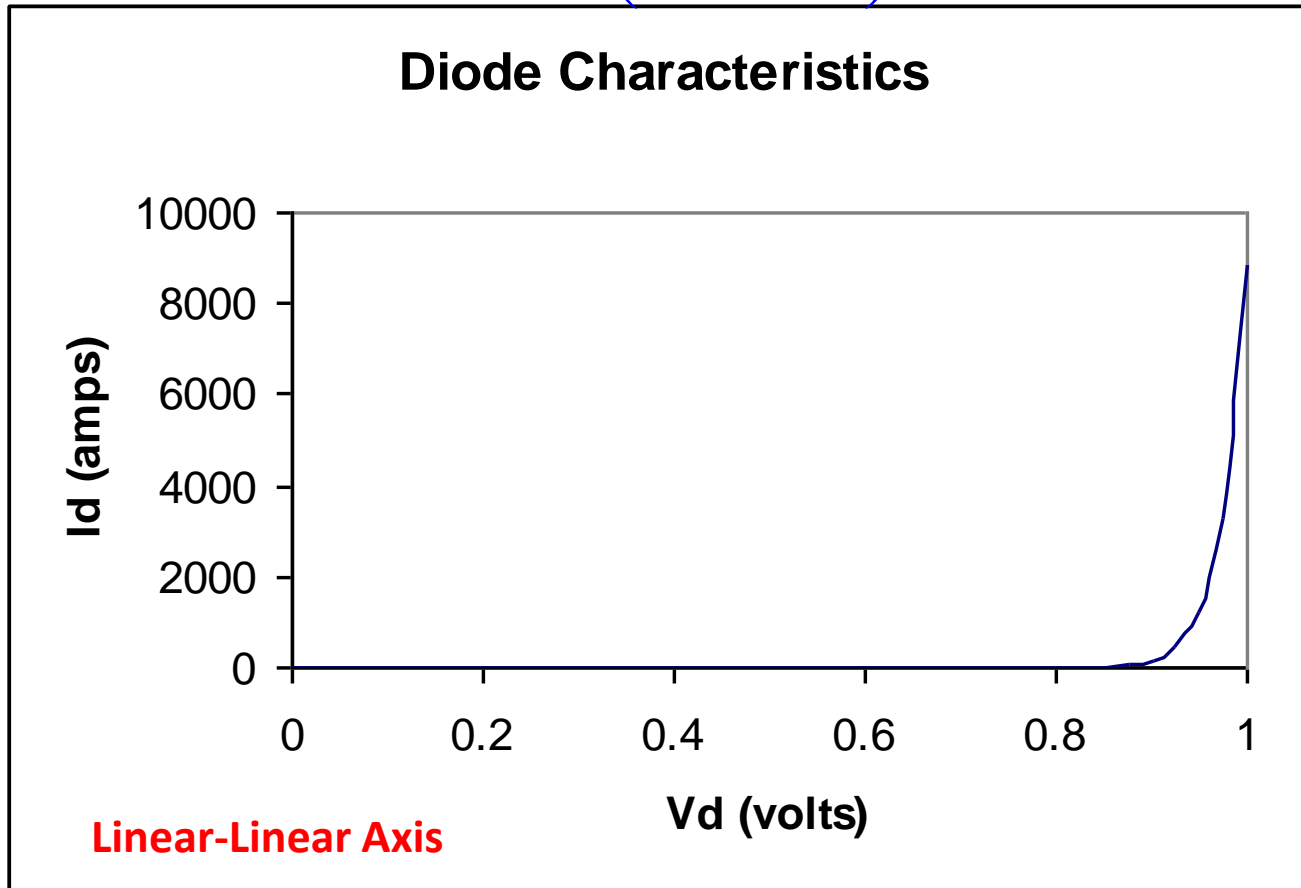
Even the simplest diode circuit does not have a closed-form explicit solution when diode equation is used to model the diode !!

Due to the nonlinear nature of the diode equation

Simplifications of diode model are essential if analytical results are to be obtained !

Lets study the diode equation a little further

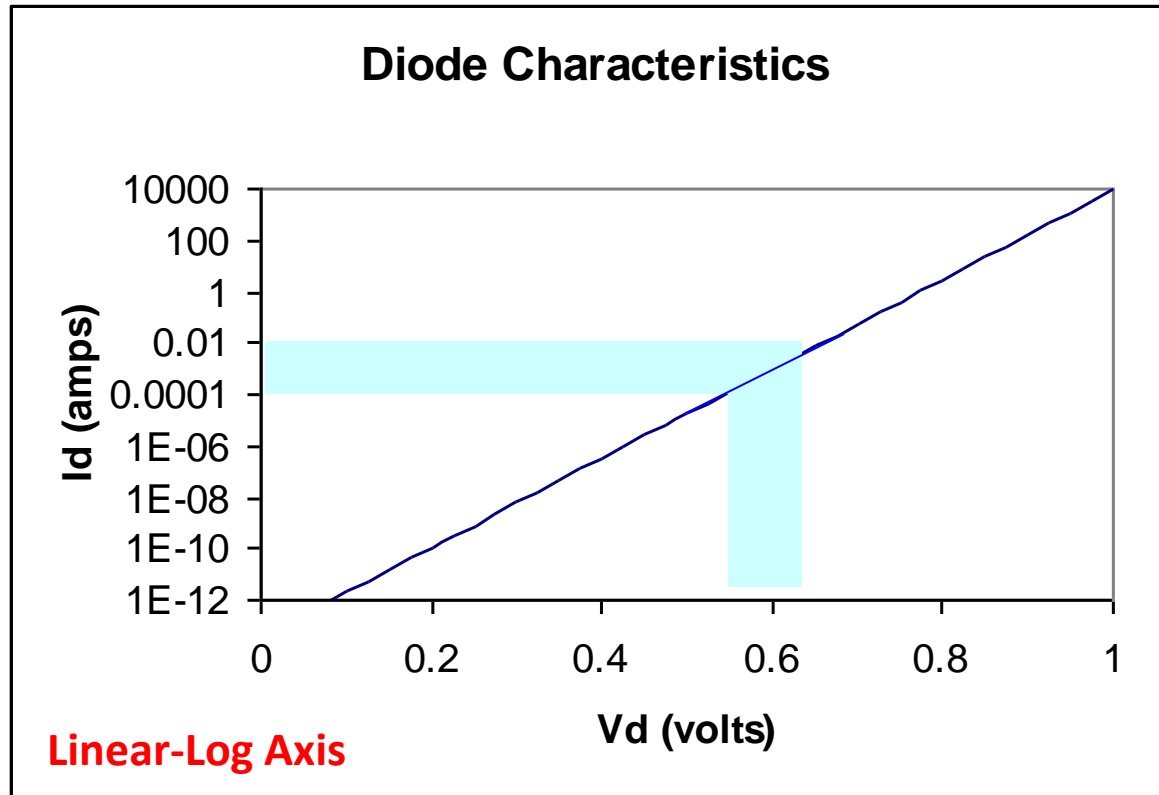
$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$



Power Dissipation Becomes Destructive if $V_d > 0.85$ V (actually less)

Lets study the diode equation a little further

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

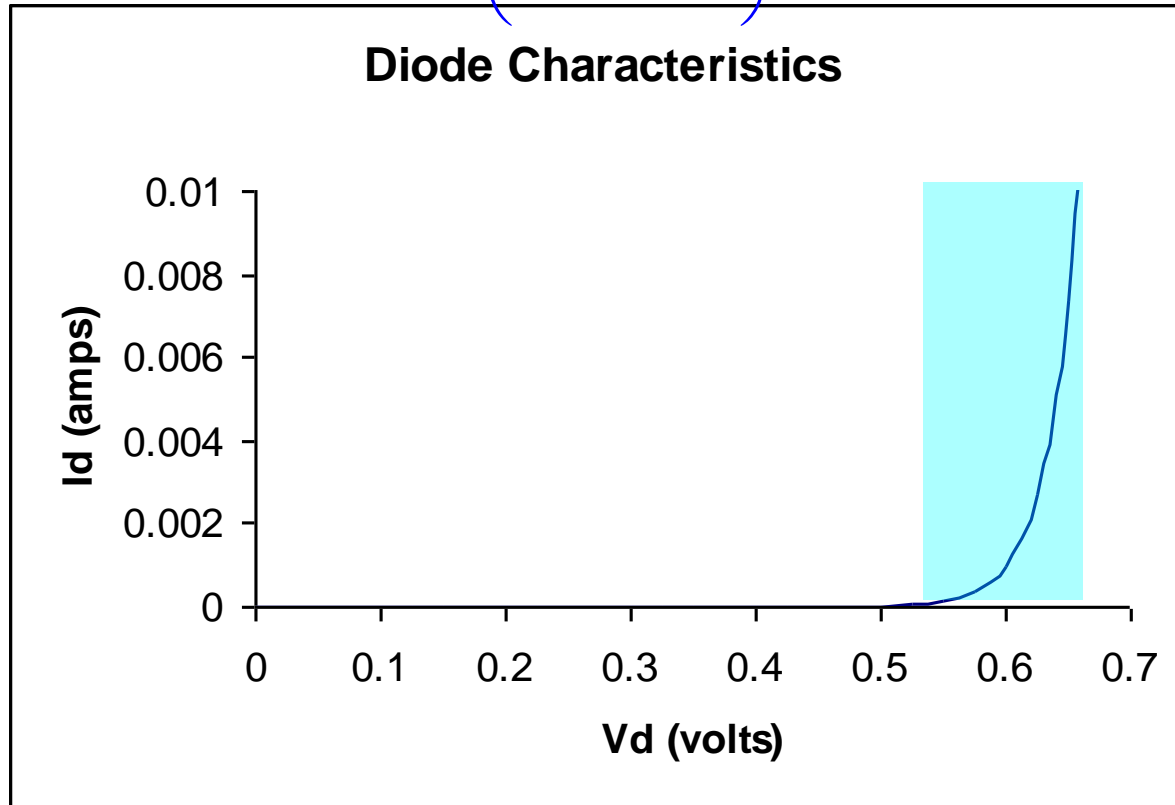


For two decades of current change, Vd is close to 0.6V

This is the most useful conducting current range for many applications

Lets study the diode equation a little further

$$I_d = I_S \left(e^{\frac{V_d}{V_t}} - 1 \right)$$



For two decades of current change, V_d is close to 0.6V

This is the most useful current range when conducting for many applications

Lets study the diode equation a little further

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

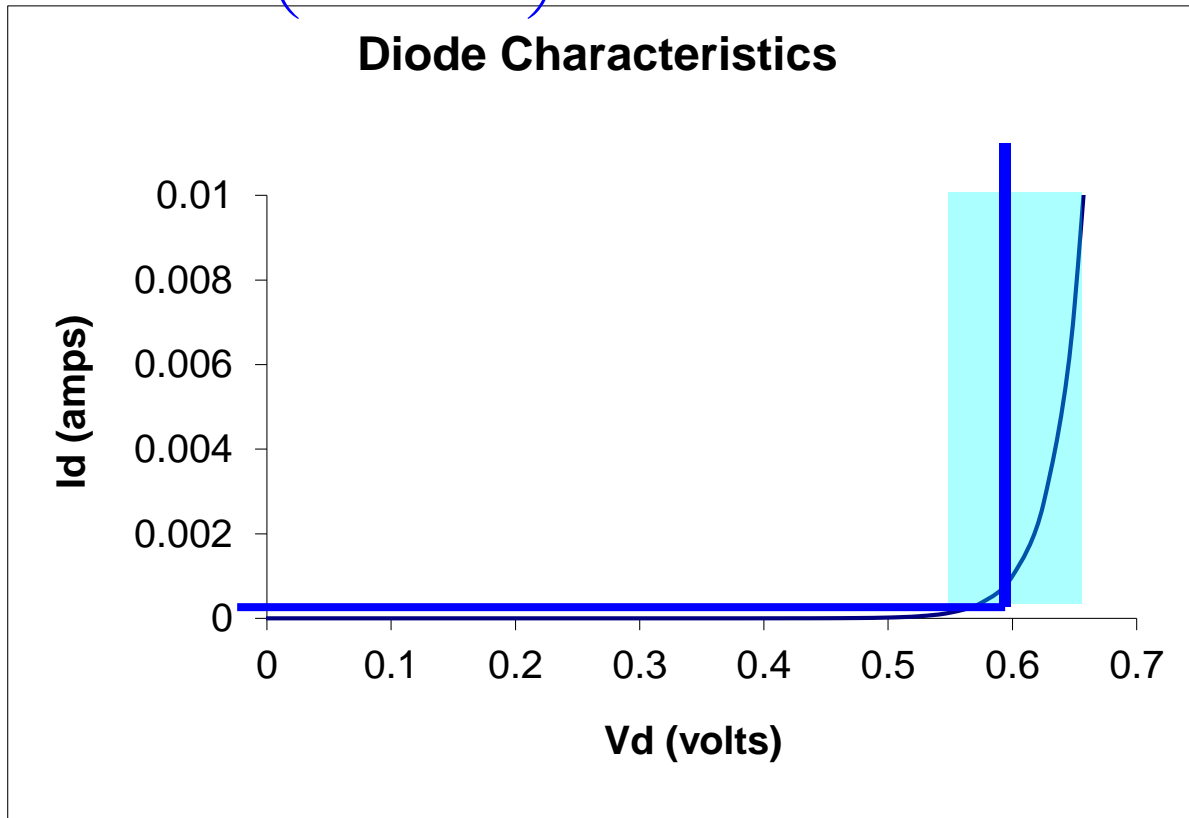


$$I_d = 0$$

$$V_d = 0.6V$$

$$V_d < 0.6V$$

$$I_d > 0$$



Widely Used Piecewise Linear Model

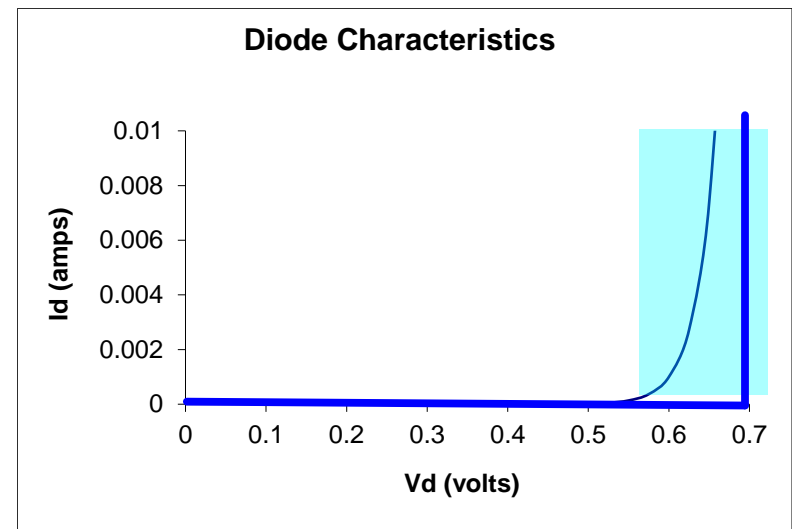
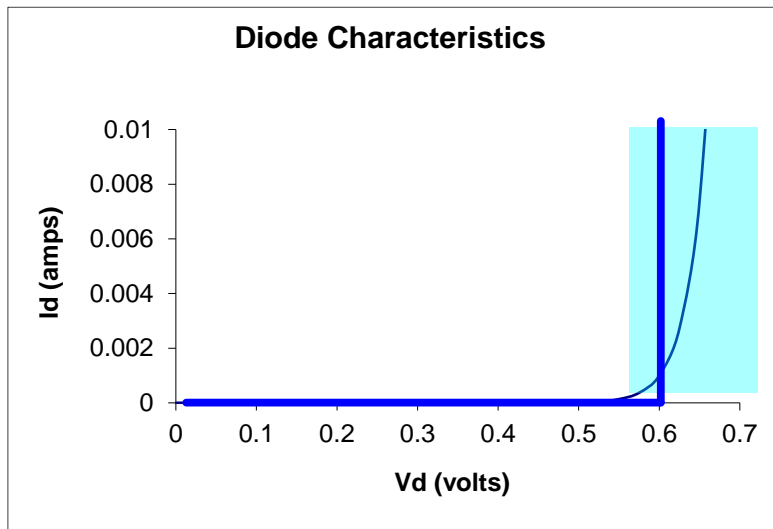
Which simplified model is better?

Both are about the same !

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

$$I_d = 0 \quad V_d < 0.6V$$
$$V_d = 0.6V \quad I_d > 0$$

$$I_d = 0 \quad V_d < 0.7V$$
$$V_d = 0.7V \quad I_d > 0$$

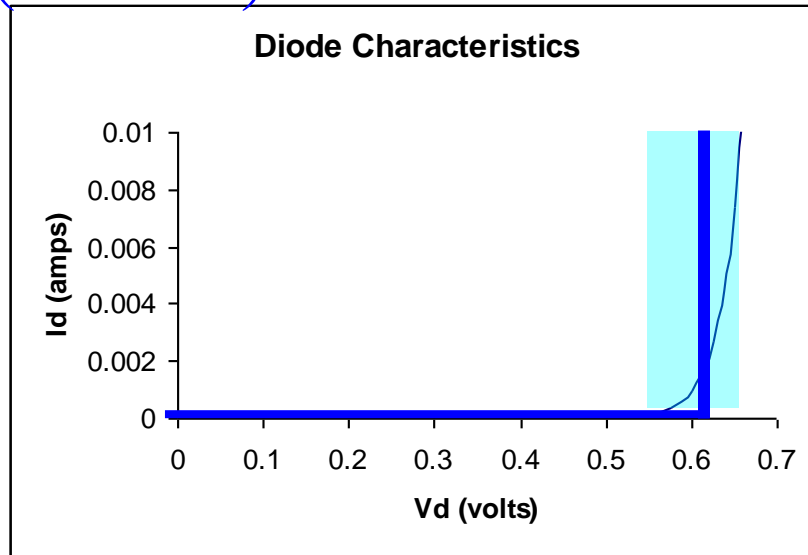


Widely Used Piecewise Linear Model

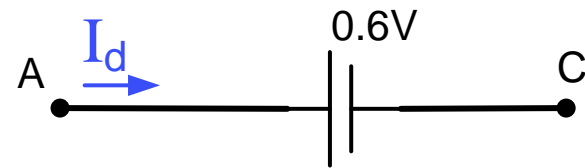
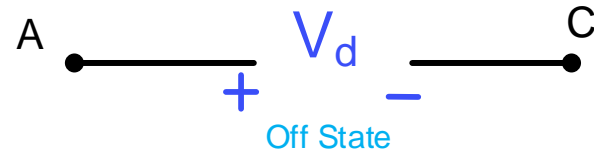
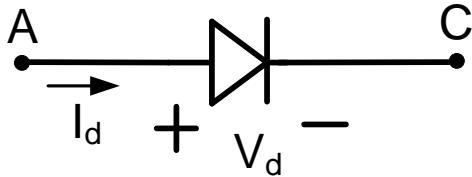
Lets study the diode equation a little further

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

\longrightarrow $I_d = 0$ $V_d < 0.6V$
 $V_d = 0.6V$ $I_d > 0$

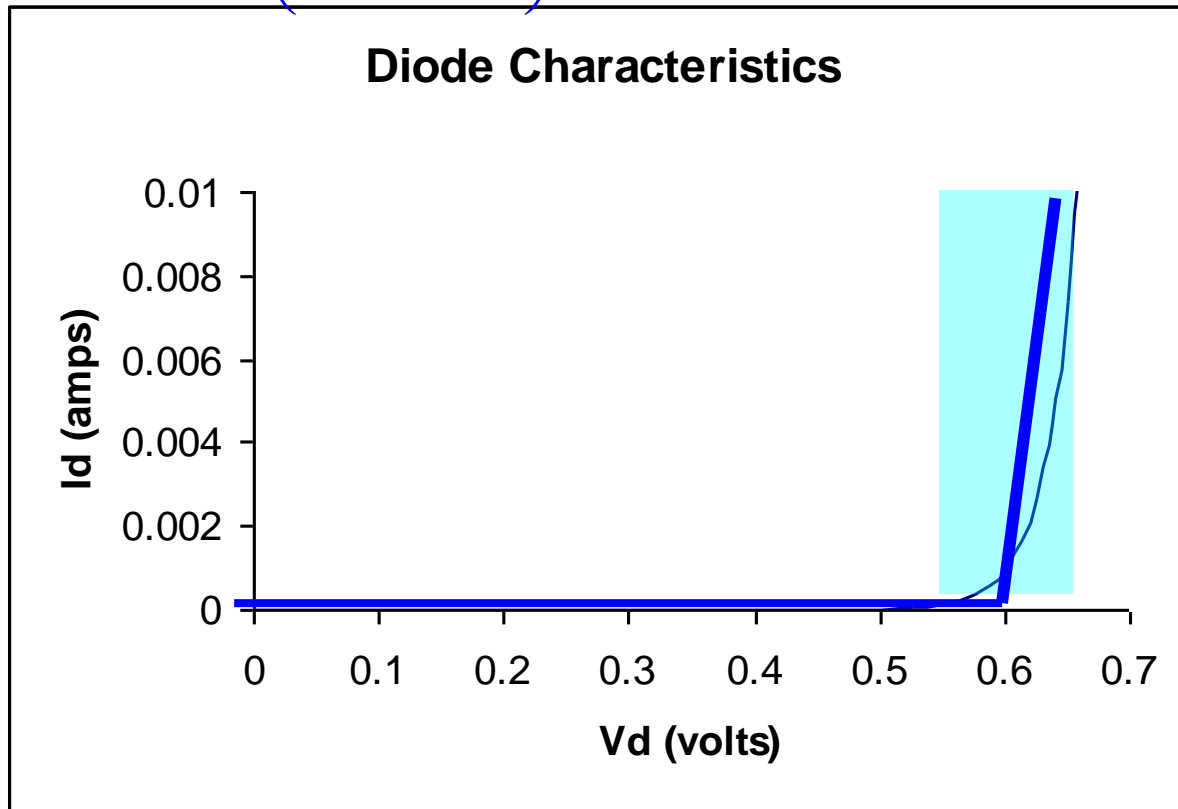


Equivalent Circuit



Lets study the diode equation a little further

$$I_d = I_s \left(e^{\frac{V_d}{V_t}} - 1 \right)$$



Better model in “ON” state though often not needed

Includes Diode “ON” resistance

Lets study the diode equation a little further

$$I_d = I_S \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

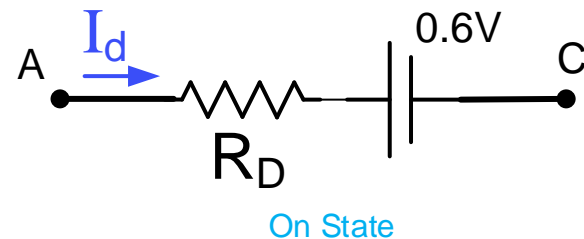
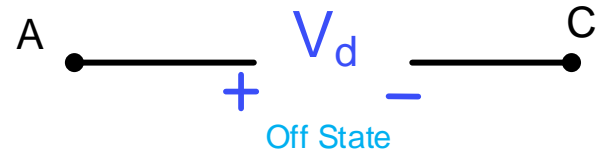
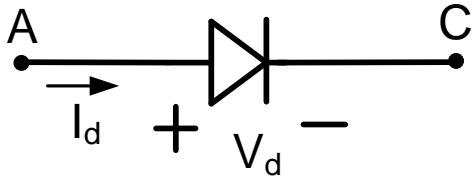
Piecewise Linear Model with Diode Resistance

$$I_d = 0 \quad \text{if } V_d < 0.6V$$

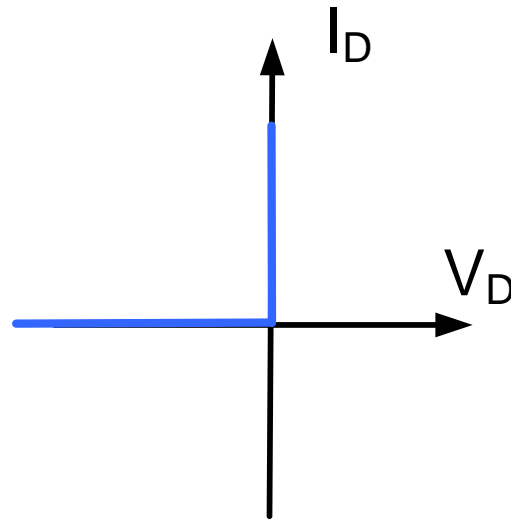
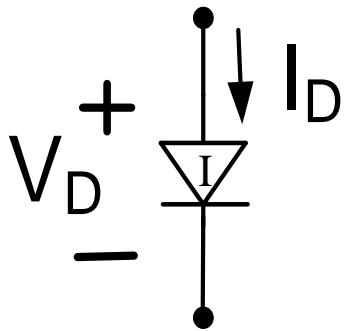
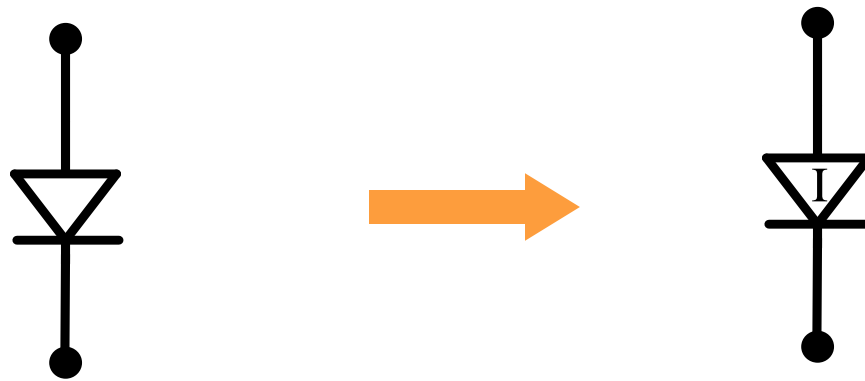
$$V_d = 0.6V + I_d R_D \quad \text{if } I_d > 0$$

(R_D is rather small: often in the 20Ω to 100Ω range):

Equivalent Circuit



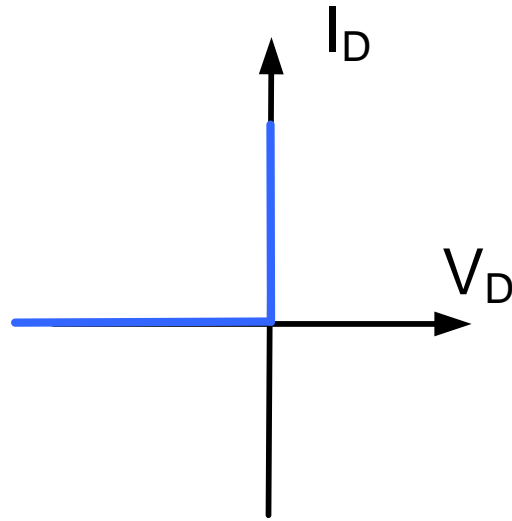
The Ideal Diode



$$I_D = 0 \quad \text{if} \quad V_D \leq 0$$

$$V_D = 0 \quad \text{if} \quad I_D > 0$$

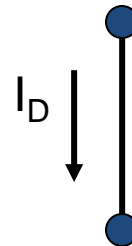
The Ideal Diode



$$I_D = 0 \quad \text{if} \quad V_D \leq 0 \quad \text{“OFF”}$$
$$V_D = 0 \quad \text{if} \quad I_D > 0 \quad \text{“ON”}$$

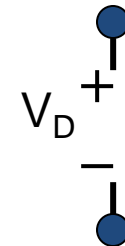


“ON”



$I_D > 0$

“OFF”

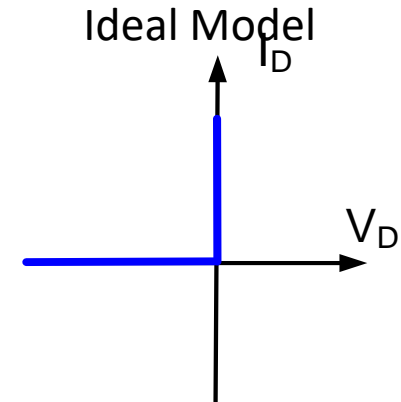
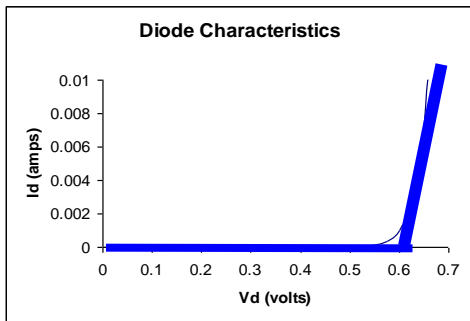
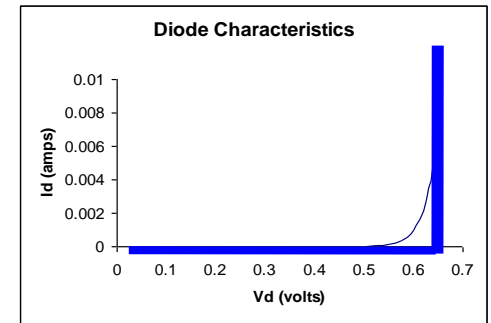
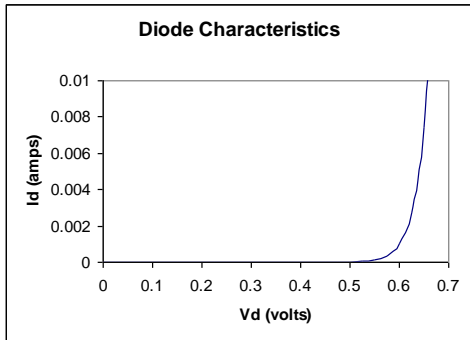


$V_D \leq 0$

Valid for

Diode Models

Diode Equation (4 variants)



Which model should be used?

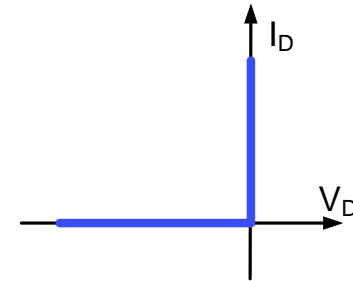
The simplest model that will give acceptable results in the analysis of a circuit

Diode Model Summary

Piecewise Linear Models

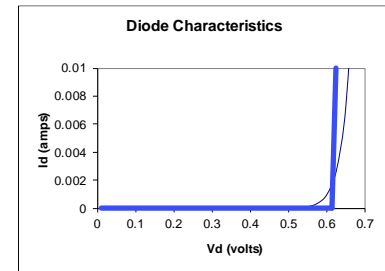
$$I_d = 0 \quad \text{if } V_d < 0$$

$$V_d = 0 \quad \text{if } I_d > 0$$



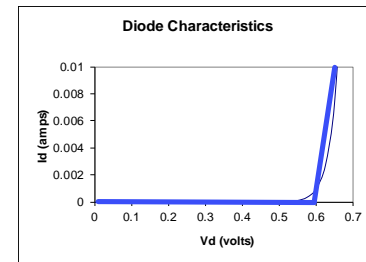
$$I_d = 0 \quad \text{if } V_d < 0.6V$$

$$V_d = 0.6V \quad \text{if } I_d > 0$$



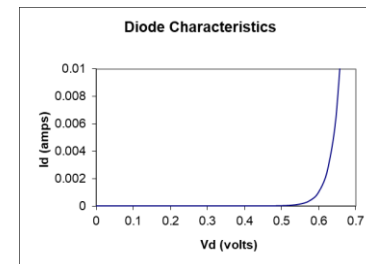
$$I_d = 0 \quad \text{if } V_d < 0.6$$

$$V_d = 0.6 + I_d R_d \quad \text{if } I_d > 0$$



Diode Equation (or variants discussed)

$$I_d = I_S \left(e^{\frac{V_d}{V_t}} - 1 \right)$$



Diode Model Summary

Piecewise Linear Models

$$I_d = 0 \quad \text{if } V_d < 0$$

$$V_d = 0 \quad \text{if } I_d > 0$$

$$I_d = 0 \quad \text{if } V_d < 0.6V$$

$$V_d = 0.6V \quad \text{if } I_d > 0$$

$$I_d = 0 \quad \text{if } V_d < 0.6$$

$$V_d = 0.6 + I_d R_d \quad \text{if } I_d > 0$$

Diode Equation (or variants discussed)

$$I_d = I_S \left(e^{\frac{V_d}{V_t}} - 1 \right)$$

When is the ideal model adequate?

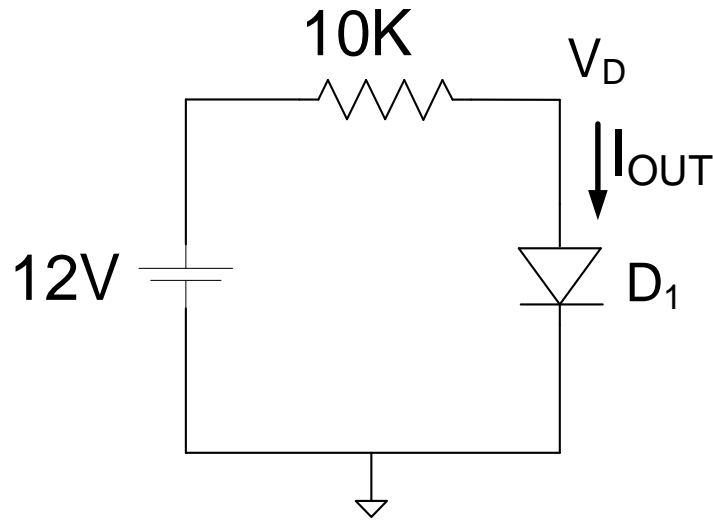
When it doesn't make much difference whether $V_d = 0V$ or $V_d = 0.6V$

When is the second piecewise-linear model adequate?

When it doesn't make much difference whether $V_d = 0.6V$ or $V_d = 0.7V$

Example:

Determine I_{OUT} for the following circuit



Solution:

If the diode equation model is used will obtain:

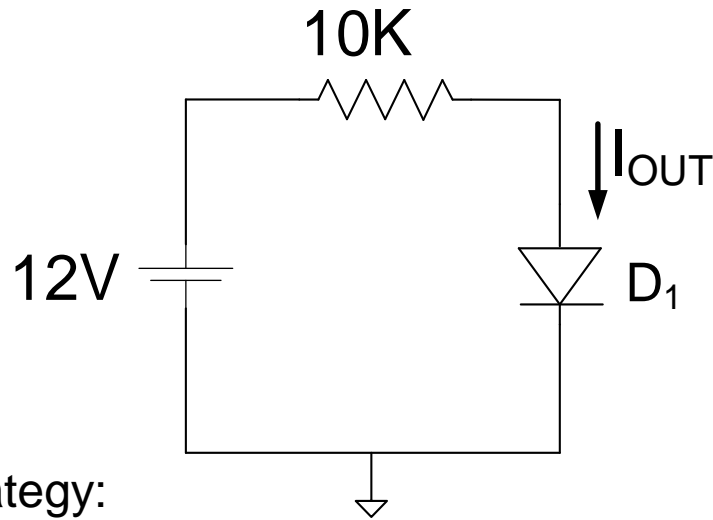
$$\left. \begin{aligned} 12 &= I_{OUT} \cdot 10K + V_D \\ I_{OUT} &= I_S \left(e^{\frac{V_D}{V_t}} - 1 \right) \end{aligned} \right\} \longrightarrow I_{OUT} = I_S \left(e^{\frac{-I_{OUT} \cdot 10K}{V_t}} e^{\frac{12}{V_t}} - 1 \right)$$

As in previous example, a closed-form explicit expression for I_{OUT} does not exist

Will now establish rigorous approach for solving this (and other) nonlinear circuit (with model uncertainty and piecewise models) and obtaining a practical solution !

Example:

Determine I_{OUT} for the following circuit



Alternate Solution Strategy:

1. Assume PWL model with $V_D=0.6V$, $R_D=0$
2. Guess state of diode (ON)
3. Analyze circuit with model
4. Validate state of guess in step 2 (verify the “if” condition in model)

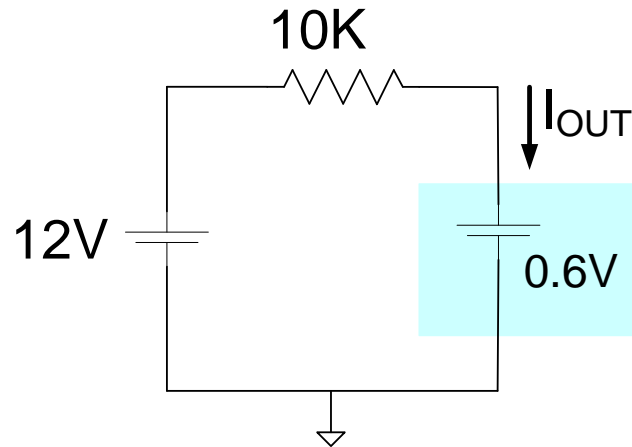
Select
Model

5. Assume PWL with $V_D=0.7V$
6. Guess state of diode (ON)
7. Analyze circuit with model
8. Validate state of guess in step 6 (verify the “if” condition in model)
9. Show difference between results using these two models is small
10. If difference is not small, must use a different model

Validate
Model

Solution:

1. Assume PWL model with $V_D=0.6V$, $R_D=0$
2. Guess state of diode (ON)



3. Analyze circuit with model

$$I_{OUT} = \frac{12V - 0.6V}{10K} = 1.14mA$$

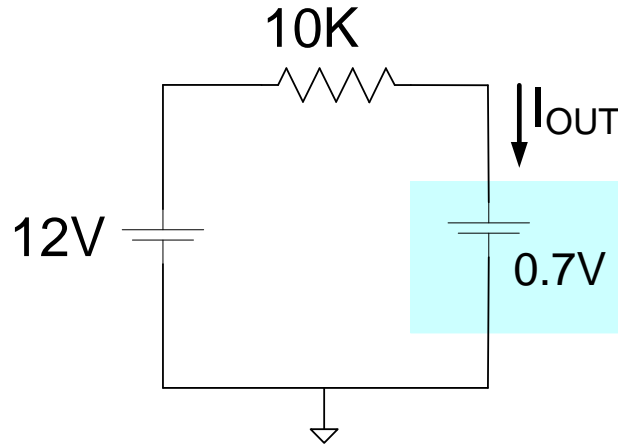
4. Validate state of guess in step 2

To validate state, must show $I_D > 0$

$$I_D = I_{OUT} = 1.14mA > 0$$

Solution:

5. Assume PWL model with $V_D=0.7V$, $R_D=0$
6. Guess state of diode (ON)



7. Analyze circuit with model

$$I_{OUT} = \frac{12V - 0.7V}{10K} = 1.13mA$$

8. Validate state of guess in step 6

To validate state, must show $I_D > 0$

$$I_D = I_{OUT} = 1.13mA > 0$$

Solution:

9. Show difference between results using these two models is small

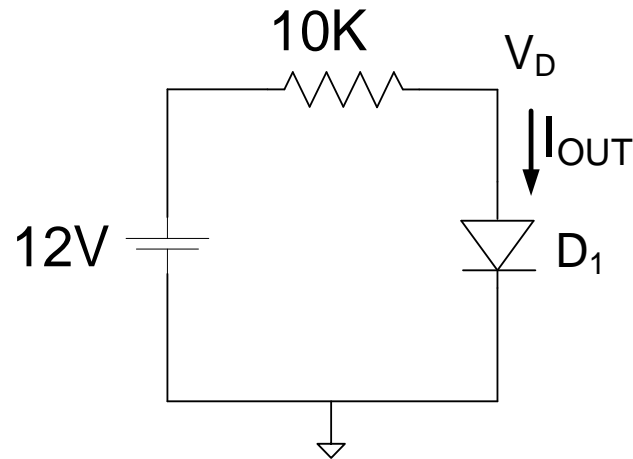
$I_{\text{OUT}}=1.14\text{mA}$ and $I_{\text{OUT}}=1.13\text{ mA}$ are close

Thus, can conclude

$$I_{\text{OUT}} \cong 1.14\text{mA}$$

Example:

Determine I_{OUT} for the following circuit



How do the two solutions compare?

With diode equation model :

$$I_{OUT} = I_S \left(e^{\frac{-I_{OUT} \cdot 10K}{V_t}} e^{\frac{12}{V_t}} - 1 \right)$$

With PWL model:

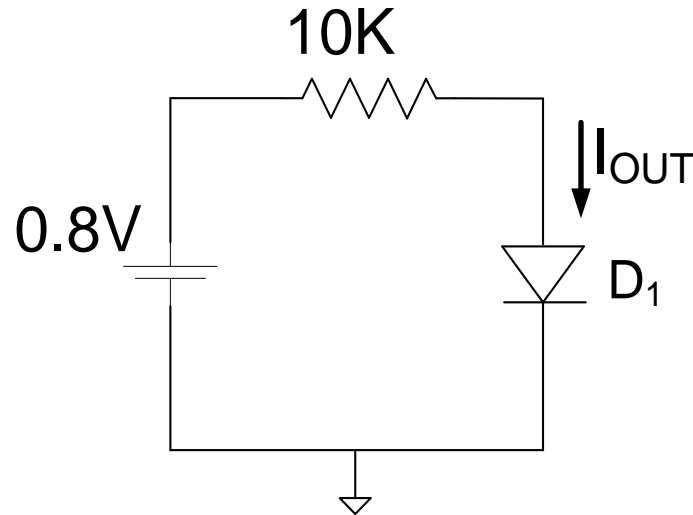
$$I_{OUT} \cong 1.14mA$$

What was the major reason the PWL model simplified the analysis?

Piecewise Linear Model

Example:

Determine I_{OUT} for the following circuit



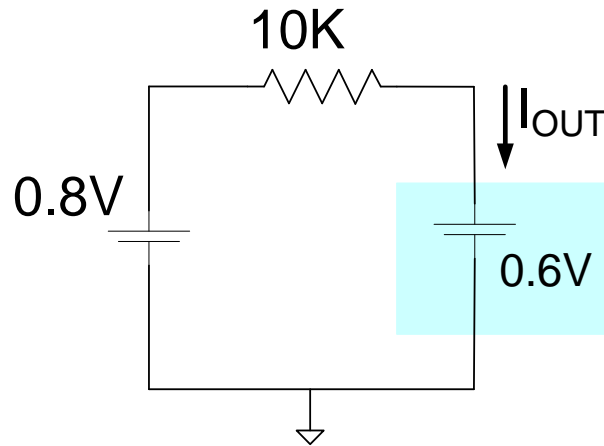
Solution:

Strategy:

1. Assume PWL model with $V_D=0.6V$, $R_D=0$
2. Guess state of diode (ON)
3. Analyze circuit with model
4. Validate state of guess in step 2
5. Assume PWL with $V_D=0.7V$
6. Guess state of diode (ON)
7. Analyze circuit with model
8. Validate state of guess in step 6
9. Show difference between results using these two models is small
10. If difference is not small, must use a different model

Solution:

1. Assume PWL model with $V_D=0.6V$, $R_D=0$
2. Guess state of diode (ON)



3. Analyze circuit with model

$$I_{OUT} = \frac{0.8 - 0.6V}{10K} = 20\mu A$$

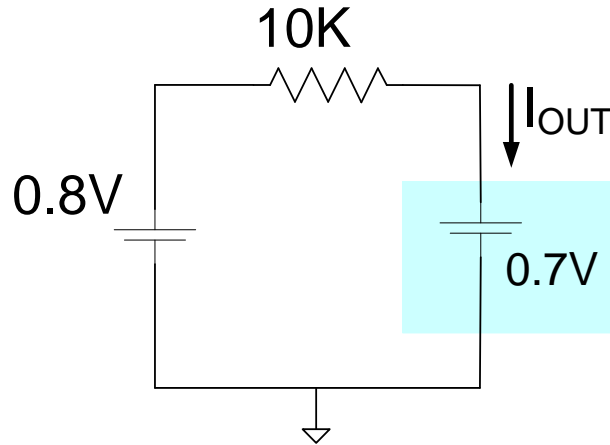
4. Validate state of guess in step 2

To validate state, must show $I_D > 0$

$$I_D = I_{OUT} = 20\mu A > 0$$

Solution:

5. Assume PWL model with $V_D=0.7V$, $R_D=0$
6. Guess state of diode (ON)



7. Analyze circuit with model

$$I_{OUT} = \frac{0.8V - 0.7V}{10K} = 10\mu A$$

8. Validate state of guess in step 6

To validate state, must show $I_D > 0$

$$I_D = I_{OUT} = 10\mu A > 0$$

Solution:

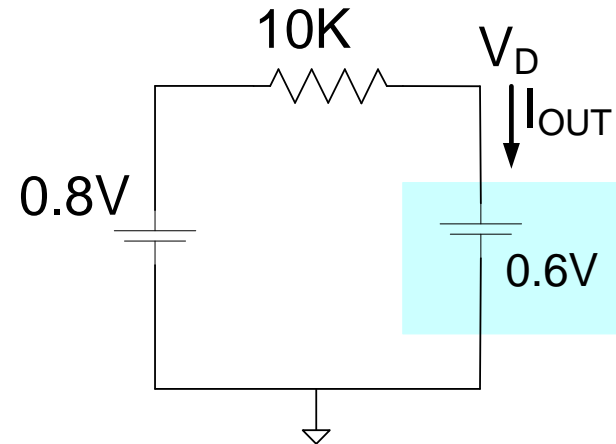
9. Show difference between results using these two models is small

$$I_{\text{OUT}}=10\mu\text{A} \text{ and } I_{\text{OUT}}=20\mu\text{A} \quad \text{are not close}$$

10. If difference is not small, must use a different model

Thus must use diode equation to model the device

$$I_{\text{OUT}} = \frac{0.8 - V_D}{10\text{K}}$$
$$I_{\text{OUT}} = I_S e^{\frac{V_D}{V_t}}$$



Solve simultaneously, assume $V_t=25\text{mV}$, $I_S=1\text{fA}$

Solving these two equations by iteration, obtain $V_D=0.6148\text{V}$ and $I_{\text{OUT}}=18.60\mu\text{A}$

Use of Piecewise Models for Nonlinear Devices when Analyzing Electronic Circuits

Process:

1. Guess state of the device
2. Analyze circuit
3. Verify State
4. Repeat steps 1 to 3 if verification fails
5. Verify model (if necessary)

Observations:

- Analysis generally simplified dramatically (particularly if piecewise model is linear)
- Approach applicable to wide variety of nonlinear devices
- Closed-form solutions give insight into performance of circuit
- Usually much faster than solving the nonlinear circuit directly
- Wrong guesses in the state of the device do not compromise solution (verification will fail)
- Helps to guess right the first time
- Detailed model is often not necessary with most nonlinear devices
- Particularly useful if piecewise model is PWL (but not necessary)
- For practical circuits, the simplified approach usually applies

Key Concept For Analyzing Circuits with Nonlinear Devices



Stay Safe and Stay Healthy !

End of Lecture 15